

# Neutrino speed anomaly as a signal of Lorentz violation

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## Abstract

The OPERA experiment just reported that the muon neutrino speed is larger than the vacuum light speed, and such result puts up a challenge to Einstein's theory of relativity and the basic principle of Lorentz invariance. We exam the possibility to attribute Lorentz violation as a source for the neutrino speed anomaly, and relate the OPERA result with Lorentz violation parameters in a new framework of standard model supplement (SMS), in which the Lorentz violation terms are brought about by a new basic principle of physical independence or physical invariance, stating that the equations describing the laws of physics have the same form in all admissible mathematical manifolds.

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The muon neutrinos from the CERN CNGS neutrino beam were detected by the OPERA detector at the Gran Sasso Laboratory through a baseline of about 730 km [1]. The neutrino velocity  $v$  is thus measured and the difference with respect to the vacuum light speed  $c$  is reported as  $(v - c)/c = (2.48 \pm 0.28(\text{stat.}) \pm 0.30(\text{sys.})) \times 10^{-5}$ . This experiment indicates that muon neutrinos propagate faster even than photons do in the vacuum. If the same experiments can be confirmed repeatedly, it would be a crucial challenge to the Einstein's relativity. The vacuum light speed is conventionally considered Lorentz invariant and the uppermost high speed of any kind of particles. Therefore the OPERA result puts up a strong challenge to the Lorentz invariance, which has been considered as one of the basic principles of modern physics.

Nowadays, there has been an increasing interest in Lorentz invariance Violation (LV or LIV) both theoretically and experimentally [2]. Among many possible ways for the realization of LV theoretically, we focus here on an attempt [3,4,5] to describe the LV effects based on a basic principle. Similar to the principle of relativity, which requires that the equations describing the laws of physics have the same form in all admissible frames of reference, we have proposed [3,4,5] a physical independence (or physical invariance) principle that the equations describing the laws of physics have the same form in all admissible mathematical manifolds. Such principle leads to the following replacement of the ordinary partial  $\partial_\alpha$  and the covariant derivative  $D_\alpha$

$$\partial^\alpha \rightarrow M^{\alpha\beta} \partial_\beta, \quad D^\alpha \rightarrow M^{\alpha\beta} D_\beta, \quad (1)$$

where  $M^{\alpha\beta}$  is a local matrix which can be divided into the sum of two matrices with  $M^{\alpha\beta} = g^{\alpha\beta} + \Delta^{\alpha\beta}$ .  $g^{\alpha\beta}$  is the metric of space-time and  $\Delta^{\alpha\beta}$  is particle-type dependent generally and a new matrix which brings new terms violating Lorentz invariance in the standard model, therefore we denote the new framework as the Standard Model Supplement (SMS) [3,4,5].

We extend SMS from the cases of Dirac particles [3] and photons [4] to the specific case of neutrinos, and then confront the OPERA result of the neutrino speed anomaly with the LV terms in the new framework. We will confirm the proportionality between the neutrino speed anomaly and the Lorentz violation parameters for the neutrino sector. Such result provides a possibility

to attribute the OPERA result as a signal for an anomalously large Lorentz violation in the neutrino sector within the new framework of SMS [3,4,5].

For the electroweak interaction sector, the Lagrangian of fermions in SMS can be written as [3]

$$\begin{aligned}\mathcal{L}_F = & i\bar{\psi}_{A,L}\gamma^\alpha\partial_\alpha\psi_{B,L}\delta_{AB} + i\Delta_{L,AB}^{\alpha\beta}\bar{\psi}_{A,L}\gamma_\alpha\partial_\beta\psi_{B,L} \\ & + i\bar{\psi}_{A,R}\gamma^\alpha\partial_\alpha\psi_{B,R}\delta_{AB} + i\Delta_{R,AB}^{\alpha\beta}\bar{\psi}_{A,R}\gamma_\alpha\partial_\beta\psi_{B,R},\end{aligned}\quad (2)$$

where  $A, B$  are flavor indices. Generally, Lorentz violation matrix  $\Delta^{\alpha\beta}$  is particle-dependent [5], so it is relevant to the flavors and has the flavor indices. For leptons,  $\psi_{A,L}$  is a weak isodoublet, and  $\psi_{A,R}$  is a weak isosinglet. After the calculation of the doublets and classification of the Lagrangian terms again, the Lagrangian can be written in a form like that of Eq. (2) too. We assume that the Lorentz violation matrix  $\Delta_{AB}^{\alpha\beta}$  is the same for the left-handedness and right-handedness, namely  $\Delta_{L,AB}^{\alpha\beta} = \Delta_{R,AB}^{\alpha\beta} = \Delta_{AB}^{\alpha\beta}$ . When we do not consider mixing between the flavor  $A$  and another flavor  $B$  for a given fermion of the flavor  $A$ , we can rewrite Eq. (2) as

$$\mathcal{L}_F = \bar{\psi}_A(i\gamma^\alpha\partial_\alpha - m_A)\psi_A + i\Delta_{AA}^{\alpha\beta}\bar{\psi}_A\gamma_\alpha\partial_\beta\psi_A, \quad (3)$$

where  $\psi_A = \psi_{A,L} + \psi_{A,R}$ , i.e., the field  $\psi_A$  is the total effects of left-handed and right-handed fermions of the given flavor  $A$ . When there is only one handedness for fermions,  $\psi_A$  is just the contributions of this one handedness. We know that neutrinos are left-handed and antineutrinos are right-handed in the standard model, therefore neutrino belongs to the case of only one handedness. We include also the mass term in the Lagrangian  $\mathcal{L}_F$ . After calculations, we can let  $m_A \rightarrow 0$  for massless fermions. Then  $\partial\mathcal{L}_F/\partial\bar{\psi}_A = 0$  gives the motion equation

$$(i\gamma^\alpha\partial_\alpha - m_A + i\Delta_{AA}^{\alpha\beta}\gamma_\alpha\partial_\beta)\psi_A = 0. \quad (4)$$

This is also the modified Dirac equation, in which the Lorentz violation term  $i\Delta_{AA}^{\alpha\beta}\gamma_\alpha\partial_\beta\psi_A$  is determined by the Lorentz violation matrix  $\Delta_{AA}^{\alpha\beta}$  of fermions of the flavor  $A$ . Multiplying  $(i\gamma^\alpha\partial_\alpha + m_A + i\Delta_{AA}^{\alpha\beta}\gamma_\alpha\partial_\beta)$  on both sides of Eq. (4) and writing it in the momentum space, we get the dispersion relation for fermions

$$p^2 + g_{\alpha\mu}\Delta_{AA}^{\alpha\beta}\Delta_{AA}^{\mu\nu}p_\beta p_\nu + 2\Delta_{AA}^{\alpha\beta}p_\alpha p_\beta - m_A^2 = 0. \quad (5)$$

When we separate the space and time components of the 4-momentum  $p$ ,

Eq. (5) reads

$$\begin{aligned}
& (1 + g_{\alpha\mu}\Delta_{AA}^{\alpha 0}\Delta_{AA}^{\mu 0} + 2\Delta_{AA}^{00})E^2 \\
& + (2g_{\alpha\mu}\Delta_{AA}^{\alpha 0}\Delta_{AA}^{\mu i} + 4\Delta_{AA}^{(0i)})Ep_i \\
& + (g^{ij} + g_{\alpha\mu}\Delta_{AA}^{\alpha i}\Delta_{AA}^{\mu j} + 2\Delta_{AA}^{ij})p_ip_j - m_A^2 = 0,
\end{aligned} \tag{6}$$

which can be simplified as

$$\alpha E^2 + \alpha^i Ep_i + \alpha^{ij} p_i p_j - m_A^2 = 0, \tag{7}$$

with the definition of the coefficients being

$$\begin{aligned}
\alpha &= 1 + g_{\alpha\mu}\Delta_{AA}^{\alpha 0}\Delta_{AA}^{\mu 0} + 2\Delta_{AA}^{00}, \\
\alpha^i &= 2g_{\alpha\mu}\Delta_{AA}^{\alpha 0}\Delta_{AA}^{\mu i} + 4\Delta_{AA}^{(0i)}, \\
\alpha^{ij} &= g^{ij} + g_{\alpha\mu}\Delta_{AA}^{\alpha i}\Delta_{AA}^{\mu j} + 2\Delta_{AA}^{ij}.
\end{aligned} \tag{8}$$

These coefficients are combinations of the elements of the Lorentz violation matrix. The velocity  $v^i$  of the fermion is the gradient of energy  $E$  with respect to the momentum  $p_i$

$$v^i \equiv \frac{\partial E}{\partial p_i} = -\frac{\alpha^i E + 2\alpha^{(ij)}p_j}{2\alpha E + \alpha^i p_i}. \tag{9}$$

Then the magnitude of  $v^i$  becomes

$$\begin{aligned}
v &\equiv \sqrt{|v_i v^i|} = \sqrt{|g_{ij} v^i v^j|} \\
&= \frac{1}{2\alpha E + \alpha^i p_i} \sqrt{|g_{ij}(\alpha^i E + 2\alpha^{(ih)}p_h)(\alpha^j E + 2\alpha^{(jk)}p_k)|},
\end{aligned} \tag{10}$$

in which the metric tensor  $g_{\alpha\beta} = \text{diag}(1, -1, -1, -1)$  and the vacuum light speed  $c = 1$ .

All of the 16 degrees of freedom of the neutrino Lorentz violation matrix are contained in Eq. (10). When we parameterize the 3-momentum  $p_i$  with the sphere coordinate system, we shall see explicitly that the velocity magnitude  $v$  in Eq. (10) is direction-dependent, and this provides the possibility for an anisotropy of the neutrino speed generally. We just focus on the neutrino speed anomaly here, so we do not consider the angle-dependence of  $v$  and discuss only a specific form  $\Delta_{AA}^{\alpha\beta} = \text{diag}(\eta, \xi, \xi, \xi)$  of the SO(3) invariant Lorentz

violation matrix. Then Eq. (10) becomes

$$v = \frac{1 - 2\xi + \xi^2}{1 + 2\eta + \eta^2} |\vec{p}|/E, \quad (11)$$

with the coefficients  $\alpha = 1 + 2\eta + \eta^2$ ,  $\alpha^i = 0$ ,  $\alpha^{ij} = (-1 + 2\xi - \xi^2)\delta^{ij}$  for Eq. (8), and the mass energy relation

$$E = \sqrt{((1 - 2\xi + \xi^2)\vec{p}^2 + m_A^2)/(1 + 2\eta + \eta^2)}.$$

So the deviation of the muon neutrino speed with respect to the vacuum light speed is

$$\frac{v - c}{c} = v - 1 = -\eta - \xi, \quad m_A \ll E. \quad (12)$$

Here Eq. (12) shows clearly that the neutrino speed anomaly is related with the Lorentz violation directly. The earlier work in Ref. [6] proposed a similar thought on the anomaly of the particle velocity due to LV. The mass energy relation Eq. (17) in Ref. [3] for the proton means that the fermion velocity  $v$  can be larger or less than the vacuum light speed  $c$ , and that the difference  $(v - c)/c$  is proportional to the magnitude  $\xi$  of the elements of the corresponding Lorentz violation matrix for protons. As for the massless gauge bosons, the difference between the photon propagating velocity  $c_\gamma$  and the Lorentz invariant light speed  $c$  is proportional to the elements of the photon Lorentz violation matrix too [4], i.e.,  $\delta c_\gamma \equiv (c_\gamma - c)/c \propto \xi$ , where  $\xi$  is now a LV parameter for photons.

We have known the neutrino velocity anomaly  $(v - c)/c$  from the latest OPERA neutrino experiment [1]

$$\frac{v - c}{c} = (2.48 \pm 0.28(\text{stat.}) \pm 0.30(\text{sys.})) \times 10^{-5}, \quad (13)$$

from which the corresponding Lorentz violation parameters of the muon neutrino  $\nu_\mu$  should satisfy

$$\eta_{\nu_\mu} + \xi_{\nu_\mu} = (-2.48 \pm 0.28(\text{stat.}) \pm 0.30(\text{sys.})) \times 10^{-5}. \quad (14)$$

From Eq. (12),  $\eta_{\nu_\mu} + \xi_{\nu_\mu}$  is negative because the velocity is larger than the vacuum light speed. Up to now, we know that the magnitude of the Lorentz violation matrix of the muon neutrino is of  $\eta_{\nu_\mu} + \xi_{\nu_\mu} \simeq 10^{-5}$ . Such order of magnitude  $10^{-5}$  for the Lorentz violation is also attained in Refs. [7,8]. In previous analysis, we get the LV parameter  $|\xi_p| \leq 10^{-23}$  for high energy protons [3]

and  $|\xi_\gamma| \leq 10^{-14}$  for photons (massless gauge bosons) [4] by confronting with relevant experimental observations. For the muon neutrino, we find that an order of magnitude  $10^{-5}$  for the LV parameters in Eq. (14) is needed to fit the OPERA speed anomaly. The electron neutrinos of the observation SN1987a also put bound on the deviation of the velocity of neutrinos  $v_{\nu_e}$  with respect to the light speed  $c$ , and from Ref. [9], it is  $|(v_{\nu_e} - c)/c| \leq 2 \times 10^{-9}$ . With Eq. (12), the constraint on the speed anomaly of the electron neutrinos means that the Lorentz violation parameters  $\eta_{\nu_e}$  and  $\xi_{\nu_e}$  satisfy  $|\eta_{\nu_e} + \xi_{\nu_e}| \leq 2 \times 10^{-9}$ . So the particle-type dependence of the Lorentz violation matrix can give a possible understanding why the electron antineutrinos of SN1987a were detected before the light by only three hours. If the OPERA speed anomaly will be confirmed again, a possible explanation of both the results of OPERA and SN1987a is that the parameters of Lorentz violation of electron neutrinos are smaller than that of muon neutrinos. At the same time, we find that the LV parameters for neutrinos with weak interaction is much larger than the corresponding LV parameters for protons and photons.

We can compare the Lorentz violation parameters from the OPERA neutrino experiment with the constraints to attribute neutrino oscillations as purely from the Lorentz violation in previous analysis [10,11]. In these analysis neutrinos are massless and neutrino flavor states are mixing states of energy eigenstates. As a consequence of Lorentz violation, different flavor states of neutrinos mix with each other when neutrinos of different eigen-energies propagate in space. From Ref. [3], the complete Lagrangian  $\mathcal{L}_F$  of fermions (cf. Eq. (3)) is

$$\begin{aligned} \mathcal{L}_F = & \bar{\psi}_A (i\gamma^\alpha \partial_\alpha - m_A) \psi_B \delta_{AB} \\ & + i\Delta_{AB}^{\alpha\beta} \bar{\psi}_A \gamma_\alpha \partial_\beta \psi_B - g\Delta_{AB}^{\alpha\beta} \bar{\psi}_A \gamma_\alpha A_\beta \psi_B, \end{aligned} \quad (15)$$

where  $A, B, \dots$  are the flavor indices,  $A_\beta$  is the gauge bosons ( $W^\pm$ ,  $Z$  for neutrinos), and  $g$  is the coupling constant. For the case of massless neutrinos, we can let  $m_A \rightarrow 0$  for the derived results. From the viewpoint of the effective field theory, the vacuum expectation value  $\langle g\Delta_{AB}^{\alpha\beta} A_\beta \rangle$  can be treated equivalently as another Lorentz violation parameter  $a_{AB}^\alpha$ , so Eq. (15) reads

$$\begin{aligned}\mathcal{L}_F &= \bar{\psi}_A(i\gamma^\alpha\partial_\alpha - m_A)\psi_B\delta_{AB} \\ &+ i\Delta_{AB}^{\alpha\beta}\bar{\psi}_A\gamma_\alpha\partial_\beta\psi_B - a_{AB}^\alpha\bar{\psi}_A\gamma_\alpha\psi_B.\end{aligned}$$

The derivative  $\partial\mathcal{L}_F/\partial\bar{\psi}_A$  leads to the motion equation

$$(i\gamma^\alpha\partial_\alpha - m_A)\psi_B\delta_{AB} + i\Delta_{AB}^{\alpha\beta}\gamma_\alpha\partial_\beta\psi_B - a_{AB}^\alpha\gamma_\alpha\psi_B = 0.$$

By multiplying  $\gamma^0$  on both sides, the terms corresponding to the operator  $i\partial_t$  belong to the Hamilton. So the Hamilton  $\mathcal{H}_{AB}$  is

$$\mathcal{H}_{AB} = -\gamma^0(i\gamma^k\partial_k - m_A)\delta_{AB} - i\Delta_{AB}^{\alpha\beta}\gamma^0\gamma_\alpha\partial_\beta + a_{AB}^\alpha\gamma^0\gamma_\alpha.$$

All the following derivations have done in Ref. [11], including the reductions of the Hamilton to quantum mechanics level and obtaining solutions of energy eigenstates. It is found that even with very small LV parameters  $\Delta_{\nu_\mu\nu_\mu}^{00} \simeq 10^{-20}$ , one can still accommodate the neutrino oscillation experiments by pure Lorentz violation effect. The parameter  $\Delta_{\nu_\mu\nu_\mu}^{00}$  is denoted by  $c_{\mu\mu}^{00}$ , and  $c_{\mu\mu}^{00} \simeq 10^{-20}$  in Ref. [11]. In this paper,  $\Delta_{AA}^{00}$  is just written as  $\eta$  and it is allowed that  $\eta_{\nu_\mu} \simeq 10^{-20}$ . Based on Eq. (14), we can get  $\xi_{\nu_\mu} \simeq 10^{-5}$  and  $|\xi_{\nu_\mu}| \gg |\eta_{\nu_\mu}|$ . Therefore the parameter  $\xi_{\nu_\mu}$  provides the main contribution to the neutrino speed anomaly from Lorentz violation, and it belongs to the space part of the muon neutrino Lorentz violation matrix. It seems that the Lorentz violation of the space components is much larger than that of the time part for the muon neutrino. Even the parameter  $\Delta_{\nu_\mu\nu_\mu}^{00}$  was chosen to be tiny at the level of  $10^{-20}$ , the neutrino oscillation experiments still can be interpreted as from pure Lorentz violation effect in Ref. [11].

Nevertheless, it should be noted also that Ref. [11] considered 3 specific forms of the tensors  $a_{AB}^\mu$  and  $\Delta_{AB}^{\alpha\beta}$  which are different from that used in this paper. We assume that  $\Delta_{AB}^{\alpha\beta}$  is diagonal here. The consistence between the OPERA results and the neutrino oscillation experiments should be further systematically studied with more general form of the Lorentz violation matrix. On the other hand, the OPERA speed anomaly still need to be examined by further experiments. There are very large degrees of freedom provided by the Lorentz violation matrices  $\Delta_{AB}^{\alpha\beta}$  for us to understand the neutrino oscillations as from Lorentz violation when the OPERA experiment is interpreted as a signal for Lorentz violation (e.g., Eq. (14) is satisfied). It is still possible that neutrinos may have small masses and that both the Lorentz violation and the conven-

tional oscillation mechanisms contribute to neutrino oscillation. In that case we have totally  $16 \times 6 + 3 = 99$  degrees of freedom, i.e., 6 Lorentz violation matrices for 3 kinds of neutrinos (when  $\Delta_{AB}^{\alpha\beta}$  has the symmetric indices  $A$  and  $B$ ) together with 3 neutrino masses, within the SMS framework to adjust parameters for confronting with relevant experimental observations.

In summary, the neutrino speed anomaly of the latest OPERA muon neutrino experiment may serve as a signal for Lorentz violation, and the OPERA result can be naturally explained by an anomalously large Lorentz violation parameter for the neutrino sector within the new framework of the standard model supplement from a basic principle of physical invariance. We thus conclude that the OPERA result opens up a door to understand neutrino properties and oscillations from a completely new standpoint beyond conventional understandings.

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- [12] Note added for 2nd version: The 1st version of this paper was submitted to arXiv on Sep.26, and it was held on by arXiv admin for two days. Compared to the 1st version, the formalism is updated and discussions are added in this 2nd version. It is shown that the seemingly contradiction between the OPERA and SN1987a results can be understood by flavor dependence of LV parameters in SMS.